

R-symmetry, supersymmetry breaking and metastable vacua in global and local supersymmetric theories

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ABSTRACT: We study $N = 1$ global and local supersymmetric theories with a continuous global $U(1)_R$ symmetry as models of dynamical supersymmetry (SUSY) breaking. We introduce explicit R-symmetry breaking terms into such models, in particular a generalized O’Raifeartaigh model. Such explicit R-symmetry breaking terms can lead to a SUSY preserving minimum. We classify explicit R-symmetry breaking terms by the structure of newly appeared SUSY stationary points as a consequence of the R-breaking effect, which could make the SUSY breaking vacuum metastable. We show that the R-breaking terms are basically divided into two categories. One of them does not generate a SUSY solution, or yields SUSY solutions that disappear in the case of supergravity when we tune a parameter so that the original SUSY breaking minimum becomes a Minkowski vacuum. We also show that the general argument by Nelson and Seiberg for a dynamical SUSY breaking still holds with a local SUSY except for a certain nontrivial case, and present concrete examples of the exception.

KEYWORDS: Supersymmetry Breaking, Supersymmetric Effective Theories, Supergravity Models.

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1. Introduction

Supersymmetric extensions of the standard model are promising candidate for the physics around TeV scale. Supersymmetry (SUSY) can stabilize the huge hierarchy between the weak scale and the Planck scale, and supersymmetric models with R-parity have the lightest superparticle as a good candidate for the dark matter. In addition, the minimal SUSY standard model realizes the unification of three gauge couplings at a scale $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV, which may suggest some underlying unified structure in the nature.

In our real world, the SUSY must be broken with certain amount of the gaugino and scalar masses. The dynamical SUSY breaking has a big predictability of the structure of such SUSY particles. It was shown by Nelson and Seiberg (NS) [1] that a global $U(1)_R$ symmetry is necessary for a spontaneous F-term SUSY breaking at the ground state of generic

models with a global SUSY. This predicts an appearance of massless Goldstone mode, R-axion, in dynamically SUSY breaking models with nonvanishing Majorana gaugino masses which breaks $U(1)_R$ symmetry.¹

Recently, it has been argued by Intriligator, Seiberg and Shih (ISS) [4] that the SUSY breaking vacuum we are living can be metastable for avoiding the light R-axion and also obtaining gaugino masses, and that such situation can be realized by a tiny size of explicit $U(1)_R$ breaking effects, whose representative magnitude is denoted by ϵ . Such explicit R-symmetry breaking terms can lead to a SUSY minimum, but such newly appeared SUSY minimum could be far away from the SUSY breaking minimum, which is found in the R-symmetric model without explicit R-symmetry breaking terms. Furthermore, such R-symmetry breaking terms would not have significant effects on the original SUSY breaking minimum, because R-symmetry breaking terms are tiny. The distance between the original SUSY breaking minimum and the newly appeared SUSY preserving minima may be estimated as $\mathcal{O}(1/\epsilon)$ in the field space. Thus, if R-symmetry breaking terms, i.e., the size of ϵ , are sufficiently small, the original SUSY breaking minimum would be a long-lived metastable vacuum.

On the other hand, an introduction of gravity into SUSY theories requires that the SUSY must be a local symmetry, i.e., supergravity. In supergravity, the structure of the scalar potential receives a gravitational correction, and also the background geometry of our spacetime is determined by the equation of motion depending upon the vacuum energy. In the above global SUSY model with metastable SUSY breaking vacuum, some fields have large vacuum values at the SUSY preserving vacuum. In such a case, supergravity effects might be sizable. Another important motivation to consider supergravity is to realize the almost vanishing vacuum energy. The global SUSY model always has positive vacuum energy at the SUSY breaking minimum. Supergravity effects could realize almost vanishing vacuum energy.

F-flat conditions have supergravity corrections. Thus, the supergravity model with global $U(1)_R$ symmetry would have different aspects from the global SUSY model. Furthermore, adding R-symmetry breaking terms would have different effects between global and local SUSY theories. Here we study in detail generic aspects of global and local SUSY theories with R-symmetry and generic behaviors caused by adding explicit R-symmetry breaking terms. That is, we reconsider the above argument for the dynamical SUSY breaking and its metastability by NS and ISS comparing global and local SUSY theories.

The important keypoint is to realize the almost vanishing vacuum energy. That is impossible in the SUSY breaking vacuum of global SUSY models, and that is a challenging issue in supergravity models. The vacuum energy may be tuned to vanish, e.g., by the constant superpotential term, which is a sizable R-symmetry breaking term. That would affect all of vacuum structure such as metastability of SUSY breaking vacua and presence of SUSY preserving vacua. Here we study this vacuum structure by using several concrete models, where we start R-symmetric models and add certain classes of R-symmetry breaking terms such that the vanishing vacuum energy is realized.

¹See for recent works on R-symmetry breaking, e.g. refs. [2–9] and references therein.

The following sections are organized as follows. In section 2, we study a structure of dynamical SUSY breaking in R-symmetric models with global SUSY. We consider the generalized O’Raifeartaigh (OR) model [10] following [4]. We introduce explicit R-breaking terms into the model and analyze in detail the newly appeared SUSY vacua as a consequence of the R-symmetry breaking effects. We also examine the stability of the original SUSY breaking vacuum under such R-breaking terms.

In section 3, we consider supergravity models with R-symmetry. We extend the argument by NS to the local SUSY theories and study the supergravity OR model. In this section, we also show a special SUSY stationary point, which does not obey the NS condition, and the associated SUSY breaking vacuum in a certain class of R-symmetric supergravity models. We introduce explicit R-breaking terms into the supergravity OR model in section 4 and classify them by the consequent SUSY solutions.

In section 5, we study the case with R-symmetry breaking terms (A-type) which might not cause a metastability of SUSY breaking minimum, because corresponding SUSY vacua disappear when we set the vacuum energy at the SUSY breaking minimum vanishing. On the other hand, in section 6, we show that another class of R-symmetry breaking terms (B-type) makes SUSY breaking minimum metastable. Section 7 is devoted to conclusions. In appendix A, we show some general features of R-axion masses, and find that the special SUSY solution exhibited in section 3 is at best a saddle point solution.

2. Global supersymmetric theory

2.1 R-symmetric model

First, we review briefly the argument by Nelson and Seiberg [1] in R-symmetric global SUSY models. Let us consider the global SUSY model with n chiral superfields z_I ($I = 1, \dots, n$) and their superpotential $W(z_I)$. In the case of global SUSY, F-flat conditions are determined by

$$W_{z_I} = 0, \tag{2.1}$$

where $W_{z_I} = \partial_{z_I} W$. Hereafter we use a similar notation for derivatives of functions $H(X)$ by fields X as H_X . The conditions (2.1) are n complex equations for n complex variables, and these can have a solution for generic superpotential.

Now, we consider global SUSY models with a continuous global $U(1)_R$ symmetry and a nonvanishing superpotential. Since the superpotential has the R-charge 2, there exists at least one field with a nonvanishing R-charge. Suppose that the n -th component z_n is such a field with the nonvanishing R-charge, $q_{z_n} \neq 0$. Then, in the following field basis

$$\begin{aligned} \chi_i &= \frac{z_i}{z_n^{q_{z_i}/q_{z_n}}}, & (q_{\chi_i} = 0), \\ Y &= z_n, & (q_Y = q_{z_n} \neq 0), \end{aligned} \tag{2.2}$$

where $i = 1, 2, \dots, n - 1$, the superpotential can be written as

$$W_{\text{NS}} = Y^{2/q_Y} \zeta(\chi_i). \tag{2.3}$$

Then the F-flat conditions (2.1) are split into two pieces,

$$(2/q_Y)Y^{2/q_Y-1}\zeta(\chi_i) = 0, \tag{2.4}$$

$$Y^{2/q_Y}\partial_{\chi_j}\zeta(\chi_i) = 0. \tag{2.5}$$

When we look for an R-symmetry breaking vacuum, $\langle Y \rangle \neq 0$, these conditions are equivalent to

$$\zeta(\chi_i) = 0, \quad \partial_{\chi_j}\zeta(\chi_i) = 0, \tag{2.6}$$

which are n complex equations for $n - 1$ complex variables, that is, these are *over-constrained* conditions. These cannot be satisfied at the same time for a generic function $\zeta(\chi_i)$, and the SUSY can be broken. This is an observation by Nelson and Seiberg [1] that the existence of an R-symmetry is the necessary condition for a dynamical SUSY breaking, and is also the sufficient condition if the R-symmetry is spontaneously broken, $\langle Y \rangle \neq 0$.

However, the scalar potential, which is obtained from the superpotential (2.3) and the Kähler potential $K(|Y|, \chi_i, \bar{\chi}_i)$, is found to have the global minimum at $Y = 0$, unless the Kähler potential $K(|Y|, \chi_i, \bar{\chi}_i)$ is non-trivial. Thus, SUSY is not broken dynamically with the NS superpotential (2.3).

The O’Raifeartaigh model [10] is a good example of R-symmetric SUSY models, where SUSY is spontaneously broken. Its generalization is shown in ref. [4] as the generalized OR model, which has the following superpotential,

$$W_{\text{OR}} = \sum_a g_a(\phi_i) X_a, \tag{2.7}$$

where $a = 1, 2, \dots, r$ and $i = 1, 2, \dots, s$, and the numbers of fields are constrained as $r > s$. Their R-charges are assigned as $q_{X_a} = 2$ and $q_{\phi_i} = 0$, and $g_a(\phi_i)$ is a function of ϕ_i . In this model, F-flat conditions for X_a are just given by

$$\partial_{X_a} W = g_a(\phi_i) = 0. \tag{2.8}$$

These are r complex equations for s complex variables, that is, these are *over-constrained* conditions for $r > s$. Therefore, there is no SUSY solution satisfying (2.8) for generic functions $g_a(\phi_i)$ with $r > s$. The superpotential of the generalized OR model (2.7) is a specific form of the NS superpotential (2.3). In the generalized OR model, SUSY is always spontaneously broken independently of whether R-symmetry is spontaneously broken or not, or the fields X_a develop nonvanishing vacuum expectation values or not.

The simplest OR model is the model with $r = 1$ and $s = 0$, and has the superpotential

$$W_{(OR)_1} = fX_1,$$

where f is a constant. Obviously, SUSY is spontaneously broken in this model, because $W_{X_1} = f$. The basic O’Raifeartaigh model corresponds to the model with $r = 2$ and $s = 1$, and $g_1(\phi) = f + \frac{1}{2}h\phi^2$ and $g_2(\phi) = m\phi$, and has the following superpotential,

$$W_{(OR)_{\text{basic}}} = \left(f + \frac{1}{2}h\phi^2 \right) X_1 + m\phi X_2. \tag{2.9}$$

The model has only a SUSY breaking pseudo-moduli space,

$$\phi = X_2 = 0, \quad X_1 : \text{undetermined}, \quad (2.10)$$

with $W_{X_1} = f$ as a global minimum of the potential. When integrating out heavy modes X_2 and ϕ , we obtain $W_{(OR)_1}$ as an effective superpotential. However, the flat direction along X_1 is lifted at the one-loop level by integrating out ϕ , and the SUSY breaking vacuum in the quantum corrected OR model is given by

$$\phi = X_2 = X_1 = 0. \quad (2.11)$$

These simple models suggest that the tadpole term of X_a is important for SUSY breaking. Indeed, we can show by simple discussion that non-vanishing terms of $g_a(\phi_i)$ at $\phi_i = 0$ are sources of SUSY breaking. We assume that $g_a(\phi_i)$ are non-singular functions. Then, we can always rewrite the superpotential (2.7) as

$$\begin{aligned} W_{\text{OR}} &= \sum_a f_a X_a + \sum_a \check{g}_a(\phi_i) X_a \\ &= \tilde{f} \tilde{X}_1 + \sum_a \tilde{g}_a(\phi_i) \tilde{X}_a, \quad (\tilde{g}_a(0) = 0), \end{aligned} \quad (2.12)$$

where $f_a = g_a(0)$, $\check{g}_a(\phi_i) = g_a(\phi_i) - f_a$, $\tilde{X}_a = U_{ab} X_b$, $\tilde{g}_a(\phi_i) = \check{g}_a U_{ab}^\dagger$ and U_{ab} is a constant unitary matrix defined by $f_a U_{ab}^\dagger = \tilde{f}_b = (\tilde{f}, 0, \dots, 0)$. In the following, we will frequently use this basis of fields and omit the tildes to simplify the notation. In this basis, the F-flat conditions for X_a , eq. (2.8), are written by

$$W_{X_a} = g_a(\phi_i) - \delta_{a1} f = 0. \quad (2.13)$$

Together with $W_{\phi_i} = \sum_a X_a \partial_{\phi_i} g_a(\phi_i) = 0$, we find that, if $f = 0$, there is a solution $X_a = \phi_i = 0$ and SUSY is not broken. Then it is obvious in the field basis (2.12) that a nonvanishing f is the source of dynamical SUSY breaking in the generalized OR model.

In the generalized OR model with the above field basis, the field X_1 plays a special role, while each of X_a ($a \neq 1$) has the qualitatively same character as others X_b ($b \neq 1$). Thus, the simple model with $r = 2$ and $s = 1$, and the superpotential,

$$W_{(OR)_2} = (f + g_1(\phi)) X_1 + g_2(\phi) X_2,$$

shows qualitatively generic aspects of the generalized OR model. Its scalar potential is written as

$$V = |f + g_1(\phi)|^2 + |g_2(\phi)|^2 + |W_\phi|^2,$$

and stationary conditions are obtained as

$$\begin{aligned} V_{X_1} &= \overline{W_\phi} g_1'(\phi) = 0, \\ V_{X_2} &= \overline{W_\phi} g_2'(\phi) = 0, \\ V_\phi &= \overline{W_\phi} W_{\phi\phi} + (\overline{f} + \overline{g_1(\phi)}) g_1'(\phi) + \overline{g_2(\phi)} g_2'(\phi) = 0, \end{aligned}$$

where $g'_a(\phi) = dg_a(\phi)/d\phi$ and $W_\phi = \sum_a X_a g'_a(\phi)$. Unless W_ϕ does not vanish, we would have *over-constrained* conditions, i.e., $g'_1(\phi) = g'_2(\phi) = 0$ for generic functions. Thus, in general, the solution of the above stationary conditions corresponds to

$$\begin{aligned} W_\phi &= X_1 g'_1(\phi) + X_2 g'_2(\phi) = 0, \\ (\bar{f} + \overline{g_1(\phi)}) g'_1(\phi) + \overline{g_2(\phi)} g'_1(\phi) &= 0. \end{aligned} \tag{2.14}$$

The latter is the condition to fix ϕ . For a fixed value of ϕ , a ratio between X_1 and X_2 is fixed by the former condition, but the linear combination

$$X_1 g'_2(\phi) - X_2 g'_1(\phi), \tag{2.15}$$

remains undetermined. That is the pseudo-flat direction, and would be lifted by loop effects. Similarly we can discuss models with several fields X_a and ϕ_i ($r > s$).

2.2 Explicit R-symmetry breaking and metastable vacua

In order to have Majorana gaugino masses in addition to soft scalar masses, the R-symmetry must be broken spontaneously or explicitly at the SUSY breaking minimum we are living. On the other hand, as shown in the previous section, the NS argument requires an exact R-symmetry for the dynamical SUSY breaking. Then, an appearance of an unwanted massless Goldstone mode, an R-axion, is inevitable in such R-symmetry breaking minimum. Does this mean the dynamical SUSY breaking is phenomenologically disfavored ?

Recently, it has been argued by Intriligator, Seiberg and Shih [4] that our world must reside in a metastable state, in order to avoid the above conflict between gaugino masses and the R-axion. The arguments are as follows. Consider a theory with an approximate R-symmetry which has a small R-symmetry breaking parameter ϵ . In the limit $\epsilon \rightarrow 0$, the R-symmetry becomes exact, and the theory possesses a SUSY breaking ground state due to the NS argument. For a nonzero but tiny parameter ϵ , this SUSY breaking minimum still remains as a local minimum of the potential, although there appear SUSY ground states somewhere in the field space due to explicit R-symmetry breaking effects. As long as the parameter ϵ is small enough, the separation between the SUSY breaking minimum and the supersymmetric vacua is large, and the former can be a long-lived metastable vacuum. These facts were exhibited by ISS based on the O’Raifeartaigh model as a simple example of dynamical SUSY breaking model with R-symmetry. Indeed, such O’Raifeartaigh-type model can be realized in some region of the moduli space of SUSY Yang-Mills theories [11].

Here following the discussion by ISS we study generic aspects of explicit R-symmetry breaking terms, and SUSY preserving vacua. We also classify explicit R-symmetry breaking terms in global SUSY models. In addition, we discuss metastability.

The simplest R-symmetry breaking term is the constant term $W_{\mathcal{R}} = c$, but the constant term does not play any role in global SUSY theory. Thus, we do not discuss about adding constant term in this section. It is obvious that when we add any R-symmetry breaking term $W_{\mathcal{R}}(Y, \chi)$ to the NS superpotential (2.3), that can relax *over-constrained* conditions and F-flat conditions can have SUSY solutions.

The generalized OR model has richer structure in explicit R-symmetry breaking terms. To see such structure, we consider the generalized OR model with three types of typical R-symmetry breaking terms, i) a function including only ϕ_i fields $W_{\mathcal{R}} = w(\phi)$, ii) a function including only X_a ($a \neq 1$), $W_{\mathcal{R}} = w(X_a)$, and iii) a function including only X_1 , $W_{\mathcal{R}} = w(X_1)$. The first type of R-symmetry breaking terms $W_{\mathcal{R}} = w(\phi)$ do not change F-flat conditions for X_a , i.e., $\partial_{X_a} W = f\delta_{a1} + g_a(\phi_i) = 0$. Hence, there is no SUSY solution.

For the second type of R-symmetry breaking terms $W_{\mathcal{R}} = w(X_a)$ ($a \neq 1$), F-flat conditions are obtained as

$$\begin{aligned} W_{X_1} &= f + g_1(\phi_i) = 0, \\ W_{X_a} &= g_a(\phi_i) + w_{X_a}(W_a) = 0 \quad \text{for } a \neq 1, \\ W_{\phi_i} &= \sum_a X_a \partial_{\phi_i} g_a(\phi_i) = 0. \end{aligned}$$

Thus, if $w_{X_a}(W_a) \neq 0$ for all of X_a , *over-constrained* conditions can be relaxed and a SUSY solution can be found. If all of ϕ_i vanish, we have $g_1(\phi_i) = 0$ and the condition $W_{X_1} = 0$ can not be satisfied. Hence, the SUSY minimum, which appears by adding $W_{\mathcal{R}} = w(X_a)$ ($a \neq 1$), corresponds to the point, where some of ϕ_i develop nonvanishing vacuum expectation values.

For the third type of R-symmetry breaking terms $W_{\mathcal{R}} = w(X_1)$, F-flat conditions are obtained as

$$\begin{aligned} W_{X_1} &= f + g_1(\phi_i) + \partial_{X_1} w(X_1) = 0, \\ W_{X_a} &= g_a(\phi_i) = 0 \quad \text{for } a \neq 1, \\ W_{\phi_i} &= \sum_a X_a \partial_{\phi_i} g_a(\phi_i) = 0. \end{aligned}$$

If $r = s + 1$, the *over-constrained* conditions can be relaxed. In this case, the point $\phi_i = 0$ for all of i can be a solution for $W_{X_a} = 0$ for $a \neq 1$. Furthermore, the conditions,

$$f + \partial_{X_1} w(X_1) = 0, \quad \sum_a X_a \partial_{\phi_i} g_a(\phi_i) = 0,$$

should be satisfied.

When R-symmetry breaking terms include X_1 and X_a ($a \neq 1$), *over-constrained* conditions can be relaxed and a solution for F-flat conditions would correspond to $\phi_i \neq 0$ for some of ϕ_i .

The SUSY breaking minimum is found in the generalized OR model without explicit R-symmetry breaking terms, as discussed in the previous subsection. As discussed above, SUSY vacua can appear, when we add the definite form of explicit R-symmetry breaking terms to the generalized OR model. Thus, the previous SUSY breaking minimum is a metastable vacuum, if such R-symmetry breaking effects are small around the SUSY breaking minimum and the SUSY breaking vacuum itself is not destabilized by such R-symmetry breaking terms.

As an illustrating example, we consider the basic OR model (2.9) with explicit R-symmetry breaking terms. ISS introduced an explicit R-symmetry breaking term in the

superpotential,² $W = W_{(OR)\text{basic}} + W_{\mathcal{R}}$, where

$$W_{\mathcal{R}} = \frac{1}{2}\epsilon m X_2^2. \tag{2.16}$$

In this case, there appears a SUSY minimum,

$$\phi = \sqrt{-\frac{2f}{h}}, \quad X_2 = -\frac{1}{\epsilon}\phi, \quad X_1 = \frac{m}{\epsilon h},$$

which is far away from the (local) SUSY breaking minimum (2.11) for a sufficiently small $\epsilon \ll 1$. In addition, the SUSY breaking minimum is not destabilized by the above R-symmetry breaking term (2.16). Then the original SUSY breaking vacuum (2.11) becomes metastable which can be parametrically long-lived for $\epsilon \ll 1$.

Instead, if we consider the following R-breaking term [13]

$$W_{\mathcal{R}} = \frac{1}{2}\epsilon m X_1^2, \tag{2.17}$$

the newly appeared SUSY point is found as

$$\phi = X_2 = 0, \quad X_1 = -\frac{f}{\epsilon m}.$$

In this case, the pseudo-moduli space (2.10) disappears at the tree level. However, the SUSY breaking point (2.11) remains as a local minimum due to the one-loop mass for X_1 , but becomes metastable. Then the situation is similar to the above example. We easily find that any R-breaking terms which consist of only ϕ do not restore SUSY.

Now, let us study whether the SUSY breaking minimum, which is found without R-symmetry breaking terms, is destabilized by adding R-symmetry breaking terms. We consider the generalized OR model with $(r = 2, s = 1)$, i.e., $W_{(OR)_2}$, whose stationary conditions (2.14) are studied in the previous subsection. Their solutions are denoted by $X_a = X_a^{(0)}$ and $\phi = \phi^{(0)}$. First, we add a small R-symmetry breaking term, $W_{\mathcal{R}} = \epsilon w(X_2)$, which depends only on X_2 . Then, the scalar potential is written as

$$V = |f + g'_1(\phi)|^2 + |g_2(\phi) + \epsilon w'(X_2)|^2 + |W_\phi|^2,$$

where $W_\phi = X_1 g'_1(\phi) + X_2 g'_2(\phi)$. In addition, we assume that the stationary conditions of V are satisfied by $X_a = X_a^{(0)} + \delta X_a$ and $\phi = \phi^{(0)} + \delta\phi$, and that all of δX_a and $\delta\phi$ are of $\mathcal{O}(\epsilon)$. For example, the stationary condition along ϕ , $V_\phi = 0$, gives the following condition,

$$\left(\sum_a |g'_a(\phi^{(0)})|^2 + \sum_a (\bar{f}\delta_{a1} + \overline{g_a(\phi^{(0)})})g''_a(\phi^{(0)}) \right) \delta\phi + \epsilon g'_2(\phi^{(0)}) \overline{w'(X_2^0)} = 0,$$

where we have used the stationary conditions (2.14) at $X_a = X_a^{(0)}$ and $\phi = \phi^{(0)}$. This is the equation to determine $\delta\phi$. The stationary condition along X_1 , $V_{X_1} = 0$, reduces to

$$g'_1(\phi^{(0)}) \overline{\delta W_\phi} = 0,$$

²See also ref. [12].

where

$$\delta W_\phi = \sum_a g'_a(\phi^{(0)})\delta X_a + \sum_a X_a^{(0)}g''_a(\phi)\delta\phi.$$

Thus, this shows a relation among δX_a and $\delta\phi$ unless $g'_1(\phi^{(0)}) = 0$. On the other hand, the stationary condition along X_2 , $V_{X_2} = 0$, leads to the following equation,

$$\epsilon w''(X_2^{(0)}) \overline{g_2(\phi^{(0)})} = 0.$$

This is not an equation among δX_a and $\delta\phi$, but implies that the stationary condition is destabilized unless $w''(X_2^{(0)}) \overline{g_2(\phi^{(0)})} = 0$. In the above basic O’Raifeartaigh model, we have $g_2(\phi^{(0)}) = 0$. Thus, the SUSY breaking minimum is not destabilized by adding the mass term of X_2 , $w(X_2) = \frac{1}{2}mX_2$, i.e., $w''(X_2) \neq 0$ at $X_2 = 0$.

Now, let us add an R-symmetry breaking term, $W_R = \epsilon w(X_1)$, which depends only on X_1 . Similarly, we can examine stationary conditions of the scalar potential,

$$V = |f + g'_1(\phi) + \epsilon w'(X_1)|^2 + |g_2(\phi)|^2 + |W_\phi|^2.$$

The stationary conditions along X_2 and ϕ give an equation to determine $\delta\phi$ and a relation among δX_a and $\delta\phi$. However, the stationary condition along X_1 , $V_{X_1} = 0$, leads to

$$w''(X_1^{(0)}) \left(\overline{f} + \overline{g_1(\phi^{(0)})} \right) = 0.$$

If this condition is not satisfied, the stationary condition at the SUSY breaking vacuum is destabilized. Indeed, the basic O’Raifeartaigh model has $f + g_1(\phi) = f$ at $\phi = 0$. Thus, when we add the mass term of X_1 , $w(X_1) = \frac{1}{2}mX_1^2$, i.e., $w'' \neq 0$, the SUSY breaking minimum become destabilized at the tree level as shown above. Note that this kind of destabilization would be related to the existence of the flat direction (2.15) in the OR model with global SUSY.³

The above discussion shows that adding generic R-symmetry breaking terms can destabilize the SUSY breaking minimum, which is found in the model without such explicit R-symmetry breaking terms. In order to realize metastability of the original SUSY breaking minimum, we need a certain type of R-symmetry breaking terms. Alternatively, loop-effects would be helpful not to destabilize the original SUSY breaking minimum by R-symmetry breaking terms.

3. R-symmetry in supergravity

In the previous section, based on the argument by ISS, we have shown that some sort of explicit R-symmetry breaking terms can restore SUSY, and the original SUSY breaking vacuum can become metastable when a certain (but not generic) class of explicit R-symmetry breaking terms are added and/or loop effects stabilize the original SUSY breaking minimum. The metastable minimum can be parametrically long-lived if the coefficient of the

³Such flat direction would be lifted by supergravity effects.

R-breaking term is sufficiently small with which the SUSY ground state is far from the metastable state in the field space.

This argument has been performed in a decoupling limit of gravity. As we find in the above discussion, however, we have to treat a large distance between some separated minima in the field space. This may imply that large vacuum values of some fields might be involved in the analysis, where supergravity effects could become sizable. Moreover, in global SUSY, the SUSY breaking minima always have a positive vacuum energy with the magnitude of the SUSY breaking scale, which never satisfies the observation that the vacuum energy almost vanishes. In such a sense, we would be forced to consider supergravity.

Note that, even in supergravity, it is often a hard task to tune the vacuum energy at the stationary points of the scalar potential to be almost vanishing. This might require a large R-symmetry breaking effect specialized to supergravity, i.e., a constant term in the superpotential [14]. The existence of such a special R-symmetry breaking term could also affect the ISS argument of metastability. Loop effects have contributions to the vacuum energy. Here we assume that such loop effects are subdominant, and we tune our parameters such that we realize $V \approx 0$ at the tree level. Hereafter we use the unit with $M_{\text{Pl}} = 1$, where M_{Pl} denotes the reduced Planck scale.

3.1 Nelson-Seiberg argument

In this subsection, we study the NS argument within the framework of supergravity theory. In the case of supergravity, F-flat conditions (2.1) are modified as

$$D_I W \equiv W_I + K_I W = 0,$$

where K denotes the Kähler potential, $K(|Y|, \chi_i, \bar{\chi}_i)$. In the field basis (2.2) with the superpotential (2.3), these are written as

$$\begin{aligned} D_{\chi_i} W &= Y^{2/q_Y} (\zeta_i + K_i \zeta) = 0, \\ D_Y W &= (2/q_Y + Y K_Y) X^{2/q_Y - 1} \zeta = 0. \end{aligned}$$

Then, we find the following two candidates of R-breaking SUSY solutions in supergravity,

$$\zeta_i = 0, \quad \zeta = 0, \tag{3.1}$$

and

$$D_{\chi_i} \zeta = \zeta_i + K_i \zeta = 0, \quad 2/q_Y + Y K_Y = 0. \tag{3.2}$$

The first conditions (3.1) contain n complex equations for $n - 1$ complex variables, and the situation is the same as the case of global SUSY (2.6), that is, the solution does not exist for a generic function ζ . On the other hand, the second conditions (3.2) are n complex equations for n complex variables which can have a solution. This corresponds to a SUSY stationary point specialized to R-symmetric supergravity.

In this subsection, we analyze the special SUSY stationary solution (3.2) which appears due to purely the supergravity effect and does not obey the NS condition. Then, in the following we assume that there is a solution for

$$2/q_Y + Y K_Y = 0. \tag{3.3}$$

For instance, if the Kähler potential is given by

$$K = \sum_{n_Y=1} c_{n_Y} |Y|^{2n_Y} + \hat{K}(\chi_i, \bar{\chi}_i), \tag{3.4}$$

the condition (3.3) becomes

$$2/q_Y + \sum_{n_Y=1} n_Y c_{n_Y} |Y|^{2n_Y} = 0.$$

Then, we need at least one negative value of $\{c_{n_Y}, q_Y\}$ to have a solution. In the simplest minimal case with $c_{n_Y>1} = 0$ (and then $K_{Y\bar{Y}} = c_1 > 0$), a negative charge, $q_Y < 0$, is required.

A nontrivial point of this solution is that this SUSY stationary point is always tachyonic as we can see from the arguments in appendix A. In addition, we can find a SUSY breaking minima along the direction $D_{\chi_i} \zeta = 0$ (the first condition in eq. (3.2)), if we assume that χ_i receives a heavy SUSY mass $m_{\chi_i}^2 \gg m_{3/2}^2$ by the condition $D_{\chi_i} W = 0$. This is a reasonable assumption because χ_i has a vanishing R-charge and $\zeta(\chi_i)$ in W is assumed to be a generic function.

The scalar potential along $D_{\chi_i} \zeta = 0$ is found to be

$$v(Y) = V \Big|_{D_{\chi_i} f=0} = e^K \left(K_{Y\bar{Y}}^{-1} |2/q_Y + K_Y Y|^2 - 3|Y|^2 \right) |Y|^{2(2/q_Y-1)} |\zeta|^2.$$

Again, for the minimal Kähler potential (3.4) with $c_1 = 1$ and $c_{n_Y>1} = 0$, the stationary condition

$$\begin{aligned} \partial_Y v(Y) &= e^{\hat{K}(\langle \chi_i \rangle, \langle \bar{\chi}_i \rangle)} e^{|Y|^2} |Y|^{2/q_Y-2} (2/q_Y + |Y|^2) \\ &\times \left(|Y|^4 + 2(2/q_Y - 1)|Y|^2 + (2/q_Y)^2 - 2/q_Y \right) = 0, \end{aligned}$$

leads to solutions

$$|Y|^2 = -2/q_Y, \tag{3.5}$$

and

$$|Y|^2 = 1 - 2/q_Y \pm \sqrt{1 - 2/q_Y}. \tag{3.6}$$

The first solution (3.5) corresponds to the SUSY saddle point and the second solutions (3.6) are SUSY breaking minima. We can find this kind of SUSY breaking minima in a similar way for more generic Kähler potential.

We can study the same system in a different view point. We redefine the field Y as

$$T = -\frac{2}{aq_Y} \ln Y, \tag{3.7}$$

where a is a real constant. In this basis, the Kähler potential and the superpotential (2.3) is written as

$$\begin{aligned} K &= K(T + \bar{T}, \chi_i, \bar{\chi}_i), \\ W &= e^{-aT} \zeta(\chi_i). \end{aligned} \tag{3.8}$$

This type of Kähler and superpotential appear in the four-dimensional effective theory derived from superstring theory, where T may be a modulus field associated to some compactified dimensions. In such a case, the Kähler potential is typically given by

$$K = -n_T \ln(T + \bar{T}) + \hat{K}(\chi_i, \bar{\chi}_i),$$

where n_T is a fractional number, and the T -dependence of the superpotential (3.8) may originate from nonperturbative effects such as string/D-brane instanton effects and gaugino condensation effects, where the corresponding gauge coupling is determined by the vacuum value of T . In this case, the scalar potential along $D_{\chi_i} \zeta = 0$ is given by

$$v(T) = V \Big|_{D_{\chi_i} \zeta = 0} = e^K \left(K_{T\bar{T}}^{-1} (K_T - a)^2 - 3 \right) |e^{-aT} \zeta|^2,$$

and then the stationary condition

$$\begin{aligned} \partial_t v(t) &= -e^{\hat{K}(\langle \chi_i \rangle, \langle \bar{\chi}_i \rangle)} e^{-at} t^{-n_T-1} \\ &\times (at + n_T) \left((a^2/n_T)t^2 + 2a(1 - 1/n_T)t + n_T - 3 \right) = 0, \end{aligned}$$

results in a SUSY saddle point $t = -n/a$ and SUSY breaking minima

$$t = -(n_T/a)(1 - 1/n_T) \pm (n_T|a|/a^2) \sqrt{5/n_T + 1/n_T^2},$$

where $t = T + \bar{T}$.

In the literature, there are examples of the models which have this kind of vacuum structure of the potential. Typical superstring models have several moduli T_I with the Kähler potential $K = \ln \prod_I (T_I + \bar{T}_I)^{-n_{T_I}}$. The superpotential induced by some nonperturbative effects is given by

$$W = \sum_n A_n e^{\sum_I a_n^I T_I},$$

where A_n and a_n^I are constants. If the number of the moduli is the same as or larger than the number of the nonperturbative terms appearing in the superpotential [15], we can define an R-symmetry. A particular linear combination of T_I 's corresponds to T in eq. (3.8) which is determined by the condition that all the remaining combinations corresponding χ_i 's receive a heavy mass by the SUSY condition $D_{\chi_i} W = 0$. This is possible for certain values of a_n^I . For the two moduli with double nonperturbative terms, i.e., racetrack models, a detailed analysis was carried out in ref. [16].

We stress that the analysis of the SUSY breaking minimum as well as the SUSY saddle point in this subsection is based on the assumption that all the other fields χ_i than Y or T are stabilized by $D_{\chi_i} W = 0$, that is, by the SUSY masses [17]. We comment that these stationary solutions have a nonvanishing and negative vacuum energy. We need to uplift the SUSY breaking minimum to a Minkowski vacuum in order to identify this minimum as the one we are living. For such purpose, we need another sector which provides the uplifting energy and is well sequestered in order not to spoil the original structure of dynamical

SUSY breaking. Such sector can be realized by a dynamically generated F-term [18, 19] for which the discussions in the following sections would be important.

In summary, there is a possibility of special SUSY stationary solution in R-symmetric supergravity with a generic superpotential. However, it is always a saddle point at best and we find SUSY breaking minima with lower vacuum energy. This may imply that the NS argument for a dynamical SUSY breaking is qualitatively correct also in this case, although there is a SUSY solution. Furthermore, the NS argument still holds in supergravity as long as the Kähler potential satisfies $2/q_Y + YK_Y \neq 0$ for any value of Y in the field basis (2.2). For instance, in typical models with $q_Y > 0$ and $K = |Y|^2$, we always find $2/q_Y + YK_Y > 0$.

3.2 Generalized O’Raifeartaigh model in supergravity

Now we consider the generalized OR model (2.7) in supergravity. The F-flat conditions (2.8) for X_a become

$$\begin{aligned} D_{X_a} W &= \partial_{X_a} W + (\partial_{X_a} K) W \\ &= \sum_b M_{ab}(X_c, \phi_i) (g_b(\phi_i) + \delta_{b1} f) = 0, \end{aligned} \tag{3.9}$$

where

$$M_{ab}(X_c, \phi_i) = \delta_{ab} + K_{X_a} X_b.$$

We define its determinant as

$$\Delta \equiv \det M_{ab} = 1 + \sum_a K_{X_a} X_a. \tag{3.10}$$

If there is no solution for $\Delta = 0$, the matrix M_{ab} has an inverse matrix and consequently the F-flat conditions (3.9) are reduced to the same ones as eq. (2.8) in the global SUSY,

$$g_a(\phi_i) + \delta_{a1} f = 0,$$

which does not allow a solution for $r > s$ in general. However, in the limit $f \rightarrow 0$ in the tilde basis (2.12), these equations are satisfied at $\phi_i = 0$. Thus, the constant f represents the typical size of SUSY breaking effects and $g_a(\phi_i)$ as the global SUSY case. We comment that the situation changes if there exists a solution of $\Delta = 0$. Actually, the condition $\Delta = 0$ is an analogue of the second condition in eq. (3.2). Then, we can carry out a similar analysis as in the previous subsection also for this OR model. That is straightforward and is omitted here. Note that the condition $\Delta = 0$ is never satisfied for a minimal Kähler potential,

$$K = \sum_a |X_a|^2 + \sum_i |\phi_i|^2. \tag{3.11}$$

In the following, we just assume that there is no solution for $\Delta = 0$.

We comment that, even in supergravity, the scalar potential is positive, $V > 0$, in the generalized OR model (2.12) with the minimal Kähler potential (3.11). In this case, the scalar potential is written as

$$V = e^K [(\bar{g}_a + \delta_{a1} \bar{f}) \{ \delta_{ab} + (|X_c|^2 - 1) \bar{X}_a X_b \} (g_b + \delta_{b1} f) + |X_a D_{\phi_i} g_a|^2].$$

For any vacuum values of X_a , we can always rotate their basis as

$$U_{ab} X_b = \hat{X}_a = (0, \dots, 0, \hat{X}_c, 0, \dots, 0),$$

by a unitary matrix $U(X_a)$, and in this basis we can write

$$e^{-K} V = \{(|\hat{X}_c|^2 - 1/2)^2 + 3/4\} |\hat{g}_c|^2 + \sum_{a \neq c} |\hat{g}_a|^2 + \sum_i |\hat{X}_c D_{\phi_i} \hat{g}_c|^2 > 0,$$

where $\hat{g}_a = (U^\dagger)_{ab} (g_b + \delta_{b1} f)$. Note that \hat{g}_a are now X_a -dependent functions. As discussed above, the conditions, $\hat{g}_a(\phi) = 0$, can not be satisfied at the same time. Thus, the vacuum energy must be positive, $V > 0$. Since typical magnitudes of $\hat{g}_a(\phi)$ would be of $\mathcal{O}(f)$, we would estimate $V \sim f^2$. To realize the almost vanishing vacuum energy $V \approx 0$ at this SUSY breaking minimum, we need a negative and sizable contribution to the vacuum energy, which can be generated by R-symmetry breaking effects, e.g., the constant term in the superpotential.

4. Explicit R-symmetry breaking in supergravity

Here we study explicit R-symmetry breaking terms in supergravity and examine whether SUSY solutions can be found by adding explicit R-symmetry breaking terms to the NS model and the generalized OR model. In the previous section, we have pointed out that there is a SUSY stationary point when the condition (3.3) or the condition $\Delta = 0$ is satisfied. In the following sections, we consider the models, where such conditions are not satisfied, and SUSY is broken in the NS and generalized OR models even within the framework of supergravity like global SUSY theory.

First we consider the NS model with explicit R-symmetry breaking terms $W_{\mathcal{R}} = w(Y, \chi_i)$. The total superpotential is written as,

$$W = Y^{2/q_Y} \zeta(\chi_i) + w(Y, \chi_i).$$

In this case, F-flat conditions of supergravity theory, $D_Y W = D_{\chi_i} W = 0$, do not lead to *over-constrained* conditions for any non-vanishing function $w(Y, \chi_i)$. It is remarkable that within the framework of supergravity theory the constant term $W_{\mathcal{R}} = c$ breaks R-symmetry and even such term is enough to relax the *over-constrained* conditions.

4.1 Generalized O’Raifeartaigh model

Let us study more explicitly the generalized OR model with explicit R-symmetry breaking terms $W_{\mathcal{R}} = w(X_a, \phi_i)$. The total superpotential is written as,

$$W = f X_1 + \sum_{a=1}^r g_a(\phi_i) X_a + w(X_a, \phi_i).$$

First, we consider the case with the constant R-symmetry breaking term, $W_{\mathcal{R}} = c$. In this case, F-flat conditions are written explicitly as

$$D_{X_a} W = f\delta_{a1} + g_a(\phi_i) + K_{X_a} \left(fX_1 + \sum_{a=1}^r g_a(\phi_i)X_a + c \right) = 0,$$

$$D_{\phi_i} W = \sum_a X_a \partial_i g_a(\phi_i) + K_{\phi_i} \left(fX_1 + \sum_{a=1}^r g_a(\phi_i)X_a + c \right) = 0.$$

The former conditions are not always *over-constrained* for $c \neq 0$. Furthermore, the vacuum expectation value of W and at least $(r - s)$ vacuum values of K_{X_a} are required to be non-vanishing. Otherwise, the former conditions become *over-constrained* for generic functions $g_a(\phi_i)$. Furthermore, when K_{X_a} for $a \neq 1$ does not vanish, all vacuum values of ϕ_i can not vanish to satisfy $D_{X_a} W = g_a(\phi_i) + K_{X_a} W = 0$. Thus, a SUSY solution can be found by adding $W_{\mathcal{R}} = c$. This solution corresponds to the AdS SUSY minimum, because non-vanishing $\langle W \rangle$ is required and the scalar potential at this point is evaluated as $V = -3e^K |W|^2 < 0$. The values of the constant c and $\langle W \rangle$ must be sizable, because this AdS SUSY point disappears in the limit that $c \rightarrow 0$ or $\langle W \rangle \rightarrow 0$. Magnitudes of c and $\langle W \rangle$ are expected to be comparable with f when $K_{X_a} = \mathcal{O}(1)$. Hence, we can find the new type of SUSY solution, which can not be found in global SUSY theory. However, that requires large values of c and $\langle W \rangle$, and may have sizable effects on the previous SUSY breaking minimum, which is found in the generalized OR model without R-symmetry breaking terms.

Similarly, we can discuss the case that R-symmetry breaking terms include only ϕ_i fields, i.e., $W_{\mathcal{R}} = w(\phi_i)$. In this case, F-flat conditions along X_a , $D_{X_a} W = 0$, are written as

$$D_{X_a} W = f\delta_{a1} + g_a(\phi_i) + K_{X_a} \left(fX_1 + \sum_{a=1}^r g_a(\phi_i)X_a + w(\phi_i) \right) = 0.$$

Thus, the situation is quite similar to the case with $W_{\mathcal{R}} = c$. To have a SUSY solution, it is required that $\langle W \rangle$, $\langle w(\phi_i) \rangle$ and at least $(r - s)$ vacuum values of K_{X_a} must be non-vanishing. Sizes of $\langle W \rangle$ and $\langle w(\phi_i) \rangle$ are expected to be comparable with f .

Finally, we consider the case that R-symmetry breaking terms include X_a fields, $W_{\mathcal{R}} = w(X_a, \phi_i)$. In this case, F-flat conditions along X_a , $D_{X_a} W = 0$, are written as

$$D_{X_a} W = f\delta_{a1} + g_a(\phi_i) + \partial_{X_a} w(X_a, \phi_i) + K_{X_a} W = 0.$$

When $K_{X_a} W$ is sufficiently small, the above F-flat conditions correspond to F-flat conditions in global SUSY theory. In such a case, we have a SUSY solution when $w(X_a, \phi_i)$ depend on at least $(r - s)$ X_a 's. Otherwise, the global SUSY solution can not be found, but a SUSY solution with $\langle w(X_a, \phi_i) \rangle \neq 0$ and $\langle W \rangle \neq 0$ can be found within the framework of supergravity theory. Such situation is similar to the case with $W_{\mathcal{R}} = c$.

We have discussed that the NS model and generalized OR model with R-symmetry breaking terms have SUSY solutions with $\langle W \rangle \neq 0$ in supergravity theory. If the SUSY breaking minimum, which is found without R-symmetry breaking terms, is not destabilized

by the presence of R-symmetry breaking terms, the previous SUSY breaking minimum would correspond to a SUSY breaking metastable vacuum. However, a sizable vacuum value of superpotential is required unless $\partial_{X_a} w(X_a, \phi_i) \neq 0$ for at least $(r - s)$ X_a fields. Such large superpotential (even if that is a constant term) would affect the stability of the previous SUSY breaking minimum.

Furthermore, we have another reason to have a large size of $\langle W \rangle$ at the previous SUSY breaking minimum. At the previous SUSY breaking minimum, the vacuum energy is estimated as $V \sim |f|^2 > 0$ for $\langle W \rangle = 0$. To realize the almost vanishing vacuum energy, $V \approx 0$, we need a non-vanishing value of $\langle W \rangle$, which are comparable with f . In this case, supergravity effects at the previous SUSY breaking minimum are not negligible. This purpose to realize $V \approx 0$ has the implication even for the case that R-symmetry breaking terms include more than $(r - s)$ X_a fields. In this case, we can find a (global) SUSY solution even for $\langle W \rangle = 0$. However, realization of $V \approx 0$ requires a sizable vacuum value of $\langle W \rangle$, although values $\langle W \rangle$ at the SUSY breaking minimum and SUSY preserving minimum are not the same. Hence, it is quite non-trivial whether one can realize a metastable SUSY breaking vacuum with $V \approx 0$ in supergravity theory, which has a SUSY minimum. We will study this possibility concretely by using simple classes of the generalized OR models in the following sections. We will concentrate ourselves to the minimal Kähler potential (3.11) in most cases of the following discussions.

4.2 Classification of R-breaking terms in supergravity

In this subsection and the following sections, we consider minutely the previous discussions about the explicit R-symmetry breaking in the supergravity framework by examining concrete examples. We introduce the explicit R-symmetry breaking terms $W_{\mathcal{R}}$ into the above supergravity OR model,

$$W_{\mathcal{R}} = c(\phi_i) + \frac{1}{2} \sum_{a,b} m \epsilon_{ab}(\phi_i) X_a X_b + \dots, \quad (4.1)$$

where $c(\phi_i)$ and $\epsilon_{ab}(\phi_i)$ are generic functions of ϕ_i including ϕ -independent constants, and the ellipsis denotes the higher order terms in X_a . Note that, as mentioned before, only the $\epsilon_{ab}(\phi_i)$ terms are relevant to the recovery of SUSY vacua in the case of global SUSY. Now we have the total superpotential, $W = W_{\text{OR}} + W_{\mathcal{R}}$. The F-flat conditions (2.13) are modified as

$$D_{X_a} W = \sum_b M_{ab} \left(g_b(\phi_i) + \delta_{a1} f_1 + \sum_{c,d} M_{bc}^{-1} \epsilon_{cd}(\phi_i) X_d + \Delta^{-1} K_{X_b} W_{\mathcal{R}} \right) = 0. \quad (4.2)$$

Here we find that all the terms in $W_{\mathcal{R}}$ including $c(\phi_i)$ are accompanied by X_a in the above F-flat conditions and then have a possibility for restoring SUSY, contrary to the case of global SUSY explained in the previous section.

Most notably, just a constant superpotential

$$W_{\mathcal{R}} = c, \quad (4.3)$$

i.e., $c(\phi_i) = c$ and $\epsilon_{ab}(\phi_i) = 0$, can restore SUSY. In this case with the minimal Kähler potential (3.11), we find a solution for eq. (4.2) as

$$\bar{X}_a = -c^{-1} \Delta g_a(\phi_i), \tag{4.4}$$

where $\Delta = 1 + \sum_a |X_a|^2$ defined in eq. (3.10) is real and positive. From eq. (4.4), X_a can be written in terms of ϕ_i , and then Δ is given by

$$\Delta = \frac{|c|^2 \pm |c| \sqrt{|c|^2 - 4 \sum_a |g_a(\phi_i)|}}{2 \sum_a |g_a(\phi_i)|},$$

which should be a real number. Therefore, in order for the SUSY solution (4.4) to be valid, the constant superpotential c must satisfy the condition

$$4 \sum_a |g_a(\langle \phi_i \rangle)|^2 \leq |c|^2, \tag{4.5}$$

where $\langle \phi_i \rangle$ are solutions of $D_{\phi_i} W = 0$ under the condition (4.4).

Because X_1 is distinguished in the superpotential (2.12), we divide the generic R-breaking terms (4.1) into two pieces:

$$W_{\mathcal{R}} = W_{\mathcal{R}}^{(A)} + W_{\mathcal{R}}^{(B)},$$

where

$$W_{\mathcal{R}}^{(A)}(X_{a \neq 1}; \phi_i) = c(\phi_i) + \frac{1}{2} \sum_{a,b \neq 1} m \epsilon_{ab}(\phi_i) X_a X_b + \dots, \tag{4.6}$$

$$W_{\mathcal{R}}^{(B)}(X_1; X_{a \neq 1}, \phi_i) = \sum_{a \neq 1} m \epsilon_{a1}(\phi_i) X_a X_1 + \frac{1}{2} m \epsilon_{11}(\phi_i) X_1^2 + \dots. \tag{4.7}$$

The ellipses denote the higher order terms in terms of $X_{a \neq 1}$ in $W_{\mathcal{R}}^{(A)}$, and those of X_1 and $X_{a \neq 1}$ in $W_{\mathcal{R}}^{(B)}$. Without loss of generality, we can assume that $\epsilon_{11}(0)$ is real and positive among $\epsilon_{ab}(0)$, which is referred as ϵ in section 6.

5. Type-A breaking: Polonyi-like models

In this section, we study the effect of R-breaking terms (4.6) which we call the A-type breaking,

$$W = W_{\text{OR}} + W_{\mathcal{R}}^{(A)}.$$

Because this type of breaking terms does not contain X_1 , we find the Polonyi model [20]

$$W|_{X_{a \neq 1}=0, \phi_i=0} = W_{\text{Polonyi}} \equiv f X_1 + c, \tag{5.1}$$

in the hypersurface $X_{a \neq 1} = 0$, $\phi_i = 0$ of the scalar potential, where $c = c(0)$. This hypersurface would be a stationary plane in the $X_{a \neq 1}$ - and the ϕ_i -directions if $\partial_{\phi_i} g_{a \neq 1}(0)$ are sufficiently large, which correspond to SUSY masses for $X_{a \neq 1}$ and ϕ_i on that plane.

Moreover, if m_1^i and/or h_1^{ij} in

$$g_1(\phi_i) = m_1^i \phi_i + h_1^{ij} \phi_i \phi_j + \dots, \quad (5.2)$$

are nonvanishing, the Polonyi model in this hypersurface can be affected/modified by a tree-level SUSY mass and/or a one-loop SUSY breaking mass for X_1 . Then, we further classify the A-type breaking models into two cases, $g_1(\phi_i) = 0$ and $g_1(\phi_i) \neq 0$.

5.1 Decoupled case: $g_1(\phi_i) = 0$

In the case with $g_1(\phi_i) = 0$, the superpotential of the A-type breaking models is written as

$$\begin{aligned} W &= fX_1 + \sum_{a \neq 1} g_a(\phi_i) X_a + c(\phi_i) + \frac{1}{2} \sum_{a,b \neq 1} m\epsilon_{ab}(\phi_i) X_a X_b + \dots \\ &= c + fX_1 + \frac{1}{2} \mu_{AB} \Phi_A \Phi_B + \dots, \end{aligned}$$

where $\Phi_A = (X_{a \neq 1}, \phi_i)$ with the index $A = (a \neq 1, i)$. The SUSY mass matrix μ_{AB} is given by the R-breaking components, $\mu_{a \neq 1, b \neq 1} = m\epsilon_{ab}(0)$, $\mu_{ij} = \partial_{\phi_i} \partial_{\phi_j} c(0)$ and the R-symmetric components, $\mu_{a \neq 1, i} = 2\partial_{\phi_i} g_a(0)$. After the unitary rotation which makes μ_{AB} diagonal, the above superpotential takes the form of

$$W = c + fX_1 + \frac{1}{2} \mu_A \Phi_A^2 + \dots, \quad (5.3)$$

where μ_A represents the eigenvalues of μ_{AB} . Because of the SUSY mass μ_A , the field Φ_A would be integrated out without affecting the low energy dynamics of X_1 , because X_1 is completely decoupled in the present case.⁴

Then, the effective action for $X \equiv X_1$ is just determined by the Polonyi superpotential (5.1), where the phase of c and f can be eliminated by the $U(1)_R$ rotation and the rephasing of X_1 . Assuming the minimal Kähler potential (3.11) for simplicity, the effective scalar potential is minimized by a real vacuum value $X = \bar{X} = x$ satisfying the stationary condition

$$V_X = e^G G_X (G_{XX} + G_X^2 - 2) = 0,$$

where $G = K + \ln |W|^2$ and

$$G_{XX} + G_X^2 - 2 = fW^{-1}(x^3 + f^{-1}cx^2 - 2f^{-1}c), \quad (5.4)$$

$$G_X = fW^{-1}(x^2 + f^{-1}cx + 1). \quad (5.5)$$

The F-flat condition for X corresponds to $G_X = 0$, and the SUSY breaking stationary point is determined by the condition $G_{XX} + G_X^2 - 2 = 0$.

As we declared, we persist in obtaining a vanishing vacuum energy at the SUSY breaking minimum. Then in addition to the stationary condition $G_{XX} + G_X^2 - 2 = 0$, we set

⁴We may have to assume that the Kähler mixing is also zero or negligible between X_1 and the others.

$V = e^G(G^{X\bar{X}}|G_X|^2 - 3) = 0$. In this case, we have to take a definite value of the constant c and find two solutions

$$(x, f^{-1}c) = (\sqrt{3} - 1, 2 - \sqrt{3}), \tag{5.6}$$

and

$$(x, f^{-1}c) = (-\sqrt{3} - 1, 2 + \sqrt{3}). \tag{5.7}$$

The mass eigenvalues of $(\text{Re}X, \text{Im}X)$ are computed as $(2\sqrt{3}f^2, (4 - 2\sqrt{3})f^2)$ for the first solution (5.6), and $(-2\sqrt{3}f^2, (4 + 2\sqrt{3})f^2)$ for the second one (5.7), at this SUSY breaking Minkowski stationary point where $W = f$. Then, only the first solution (5.6) can be a minimum of the potential, while the second one (5.7) is a saddle point. We comment that ϕ_i and $X_{a \neq 1}$ directions would not possess tachyonic masses at these points for sufficiently large SUSY mass μ_A compared with the SUSY breaking mass f . Therefore, the candidate for our present universe, where the SUSY is broken with (almost) vanishing vacuum energy, is the first solution (5.6).

In addition to a SUSY breaking solution satisfying $G_{XX} + G_X^2 - 2 = 0$, we have a SUSY solution $G_X = 0$ due to the R-breaking effect $c \neq 0$, that is,

$$x_{\pm} = \frac{1}{2}(-f^{-1}c \pm \sqrt{(f^{-1}c)^2 - 4}), \tag{5.8}$$

if the R-breaking constant c satisfies

$$|f^{-1}c| \geq 2. \tag{5.9}$$

Note that this condition (5.9) corresponds to eq. (4.5) in the previous general argument for the generalized OR model. The mass eigenvalues of $(\text{Re}X, \text{Im}X)$ are computed as $W_{\pm}^2(x_{\pm}^2 - 2)(x_{\pm}^2 + 1)$ and $W_{\pm}^2(x_{\pm}^2 - 1)(x_{\pm}^2 + 2)$ at this SUSY AdS stationary point where

$$|W_{\pm}| = |fx_{\pm} + c| = \frac{1}{2} \left| f(f^{-1}c \pm \sqrt{(f^{-1}c)^2 - 4}) \right| > 0,$$

and then we obtain

$$V = -3e^G = -3e^{x_{\pm}^2}|W_{\pm}|^2 < 0.$$

Remark that, in the vanishing (one of) R-breaking limit, $c \rightarrow 0$, the condition (5.9) is not satisfied, and the SUSY solution (5.8) disappears. In the other words, this SUSY solution is a consequence of the R-breaking constant term c in the superpotential. Due to the appearance of this SUSY solution, there is a possibility that the SUSY breaking point determined by $G_{XX} + G_X^2 - 2 = 0$ becomes a metastable vacuum as in the case of global SUSY explained previously.

However, this is not the case. Interestingly, if we tune the R-breaking constant superpotential c as $f^{-1}c = 2 - \sqrt{3}$ so that the solution (5.6) with the vanishing vacuum energy is realized, the condition (5.9) is not satisfied and the SUSY stationary solution (5.8) disappears. In such a sense, the constant R-breaking term c does not lead to a metastability of SUSY breaking Minkowski minimum (5.6).

Next, we consider the SUSY stationary solutions outside the Polonyi slice $X_{a \neq 1} = 0$, $\phi_i = 0$. For the superpotential (5.3), the F-flat directions are determined by

$$\begin{aligned} D_{\Phi_A} W &= K_A W + \mu_A \Phi_A + \dots = 0, \\ D_{X_1} W &= f + K_{X_1} W = 0, \end{aligned}$$

which can be satisfied by distinguishing a single field $\Phi_B \neq 0$ for $\exists B$ as

$$\begin{aligned} W &= -K_B^{-1} \Phi_B (\mu_B + \dots) = -K_{X_1}^{-1} f \quad (\text{for } \exists B), \\ \Phi_A &= K_A = 0 \quad (\text{for } A \neq B), \end{aligned} \quad (5.10)$$

where the ellipsis represents the higher order terms of Φ_B . The first line gives two complex equations for two complex variables X_1 and $\Phi_{\exists B}$, which have a solution in general.

For example, if the Kähler potential is minimal (3.11), all the parameters in the superpotential are real and there is no higher order terms of Φ_B (no ellipses in the above expressions), then the solution for eq. (5.10) is found as

$$|\Phi_B|^2 = -2 \left(\frac{c}{\mu_B} + \frac{f^2}{\mu_B^2} + 1 \right) > 0, \quad \Phi_{A \neq B} = 0. \quad (5.11)$$

For this value of Φ_B , the remaining condition $D_{X_1} W = 0$ is satisfied by

$$X_1 = f/\mu_B.$$

Note that the number of this SUSY points is $n_X + n_\phi - 1$ because the solution (5.11) is valid for every choice of $B = (b \neq 1, j)$. In order for the solution (5.11) to be valid, the parameter μ_B must satisfy

$$\mu_B^2 + c\mu_B + f^2 \leq 0.$$

This leads to the same condition (5.9) for the R-breaking constant term c as in the Polonyi-type SUSY solution.

In summary, the A-type breaking terms (4.6) can restore SUSY in the generalized OR model (2.7) or equivalently (2.12) in general. However, if we tune the R-breaking constant term in the superpotential so that the SUSY breaking minimum has a vanishing vacuum energy, i.e., (5.6), the SUSY solutions (5.8) and (5.11) disappear. Therefore, in this sense, the A-type R-symmetry breaking terms do not lead to a metastability of the SUSY breaking (Minkowski) vacuum aside from a possibility of the existence of more complicated SUSY solutions than (5.11).

5.2 Generic case: $g_1(\phi_i) \neq 0$

Now we turn on a nonvanishing $g_1(\phi_i)$ as in eq. (5.2). With this term, the tree-level (field dependent) mass matrices in the $\phi_i = 0$ plane contain the following contributions,

$$\begin{aligned} V_{X_1 \bar{X}_1} |_{\phi_i=0} &= |m_1^i|^2 + \dots, \\ V_{\phi_i \bar{\phi}_j} |_{\phi_i=0} &= m_1^i \bar{m}_1^{\bar{j}} + 4h_1^{ik} \bar{h}_1^{\bar{j}k} |X_1|^2 + \dots, \\ V_{\phi_i \phi_j} |_{\phi_i=0} &= h_1^{ij} \bar{f} + \dots, \\ V_{X_1 \phi_i} |_{\phi_i=0} &= 2h_1^{ij} \bar{m}_1^{\bar{j}} X_1 + \dots, \end{aligned} \quad (5.12)$$

where the ellipses represent the original terms involving $X_{a \neq 1}$, those coming from $c(\phi_i)$, and the supergravity corrections. Here the doubled indices are summed up. The Kähler covariant derivatives of the superpotential in the hypersurface $\phi_i = 0$, $X_{a \neq 1} = 0$ are given by

$$D_{X_1} W|_{\phi_l=0} = f + K_{X_1} W, \quad D_{X_{a \neq 1}} W|_{\phi_l=0} = 0, \quad D_{\phi_i} W|_{\phi_l=0} = m_1^i X_1.$$

From the third equation, we find that ϕ_i can not be integrated out prior to X_1 by the F-flat condition $D_{\phi_i} W = 0$ unlike before. This is because, with the nonvanishing m_1^i , the source field X_1 for SUSY breaking shares a common SUSY mass with ϕ_i as shown in eq. (5.12).

In this case, the purely X_1 -direction is no longer special in the scalar potential. We have to treat X_1 and ϕ_i at the same time. The analysis is quite complicated, and then we consider the case with $m_1^i = 0$ in the following, where $g_1(\phi_i)$ starts from the quadratic term in ϕ_i , and the ϕ_i can be integrated by their F-flat conditions $D_{\phi_i} W = 0$ resulting $\phi_i = 0$. We will comment about the case with $m_1^i \neq 0$ in section 6.2 together with more general R-breaking terms. The components of the mass matrices (5.12) are now reduced to

$$V_{\phi_i \bar{\phi}_j}|_{\phi_l=0} = 4h_1^{ik} \bar{h}_1^{\bar{j}k} |X_1|^2 + \dots, \quad V_{\phi_i \phi_j}|_{\phi_l=0} = h_1^{ij} \bar{f} + \dots.$$

From the second equation, we observe that some linear combinations of $\text{Re } \phi_i$ and $\text{Im } \phi_j$ become tachyonic in the $\phi_i = 0$ plane if $|h_1^{ij} f|$ dominate the SUSY mass for ϕ_i . The X_1 -dependence in the first one indicates that a SUSY breaking mass of X_1 is generated at the one-loop level, which is proportional to h_1^{ij} .

Therefore, the effective potential after integrating out ϕ_i and $X_{a \neq 1}$ is given by

$$V = V^{(0)} + V^{(1)}, \quad V^{(0)} = e^G (G^{X\bar{X}} |G_X|^2 - 3), \quad V^{(1)} = m_X^2 |X|^2, \quad (5.13)$$

where $X \equiv X_1$, $G = K + \ln |W|^2$, and the effective superpotential $W = W_{\text{Polonyi}}$ is shown in eq. (5.1). The one-loop mass m_X is determined by h_1^{ij} as well as f , which would be considered as an independent parameter in the effective action. The stationary condition $V_X = 0$ results in [19]

$$X \simeq 2fc/m_X^2,$$

for $c \sim f \sim m_X \ll 1$ in the unit with $M_{Pl} = 1$, and the vanishing vacuum energy at this minimum requires

$$c = f/\sqrt{3} + \mathcal{O}(f^3/m_X^2).$$

The SUSY is broken at this Minkowski minimum with $D_X W = f + \mathcal{O}(f^2)$ and $W = f/\sqrt{3} + \mathcal{O}(f^2)$.

6. Adding type-B breaking: Metastable universe

In the previous section, we have analyzed the generalized OR model with the explicit R-symmetry breaking terms (4.7) which do not involve the source field X_1 for the dynamical SUSY breaking.

In this section, we study more general case with the R-breaking terms (4.6) including X_1 , i.e.,

$$W = W_{\text{OR}} + W_{\mathbb{R}}^{(A)} + W_{\mathbb{R}}^{(B)}.$$

In the type-B breaking terms (4.6), the first term with $\epsilon_{a \neq 1, 1}(0)$ gives the common SUSY mass for X_1 and $X_{a \neq 1}$ in the $\phi_i = 0$ plane. Then the situation is similar to the case with a nonvanishing m_1^i in eq. (5.2), that is, we can not integrate out $X_{a \neq 1}$ prior to X_1 , and we will include this case also in section 6.2.

By setting $\epsilon_{a \neq 1, 1}(0) = 0$, the superpotential in the hypersurface $\phi_i = X_{a \neq 1} = 0$ is given by

$$W = fX + \frac{1}{2}m\epsilon X^2 + c + \dots, \quad (6.1)$$

where $X \equiv X_1$, $\epsilon = \epsilon_{11}(0)$ and the ellipsis stands for the higher order terms in X .

6.1 Decoupled case: $g_1(\phi_i) = 0$

As in the previous section, we first consider the case with $g_1(\phi_i) = 0$, where X_1 is decoupled from the others in the superpotential. In this case the hypersurface $\phi_i = X_{a \neq 1} = 0$ would be stable in the ϕ_i -, $X_{a \neq 1}$ -direction as in section 5.1. The effective theory in this slice is described by the superpotential (6.1).

With the minimal Kähler potential (3.11), real parameters f , c , m and no higher order terms (ellipsis) in the superpotential (6.1) for simplicity, the SUSY breaking and SUSY stationary conditions are respectively given by eqs. (5.4) and (5.5). In the limit $\epsilon \rightarrow 0$ of eq. (6.1), the SUSY breaking solution is given by eq. (5.6). Then we can find the deviation of X from this point assuming $\epsilon \ll 1$ and $m \sim c^{1/3} \sim f^{1/2}$. We find a SUSY breaking minimum with a vanishing vacuum energy at

$$X_{\text{SB}} = X_0 + \delta X, \quad X_0 = \sqrt{3} - 1, \quad \delta X = -\frac{\epsilon m}{2f} + \mathcal{O}(\epsilon^2), \quad (6.2)$$

where the constant superpotential term c is tuned as

$$c = (2 - \sqrt{3})f + (2\sqrt{3} - 3)\epsilon m + \mathcal{O}(\epsilon^2). \quad (6.3)$$

On the other hand, a SUSY solution,

$$X_{\text{SUSY}} \simeq -\frac{2f}{\epsilon m}, \quad (6.4)$$

arises as a consequence of the B-type R-breaking term represented by the parameter ϵ , although the vacuum energy is set to be vanishing at the SUSY breaking minimum. This is unlike the case of SUSY solutions (5.8) and (5.11) caused by the introduction of A-type R-breaking terms (4.6). The shift of SUSY breaking minimum δX in eq. (6.2) is rewritten as

$$\delta X/X_0 \simeq \frac{1}{\sqrt{3} - 1} \frac{1}{X_{\text{SUSY}}},$$

and we find

$$|X_{\text{SUSY}}| > \frac{1}{\sqrt{3} - 1} \sim \mathcal{O}(1),$$

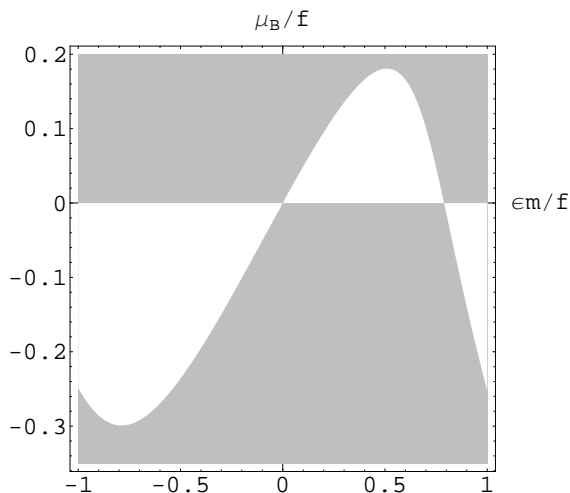


Figure 1: Parameter region (white) of μ_B and ϵ allowing the SUSY solution (6.5). All the parameters are assumed to be real and the constant term c is fixed by the vanishing vacuum energy condition (6.3) at the SUSY breaking minimum (6.2). In the shaded region, the SUSY solution (6.5) is not allowed and the SUSY breaking solution (6.2) does not become metastable due to the R-breaking effect parameterized by ϵ . We find no allowed region in the limit $\epsilon \rightarrow 0$ which corresponds to the solution (5.11).

in order for the shift δX to reside in a perturbative region, $|\delta X/X_0| < 1$.

This means that the vacuum value of $|X|$ at the newly appeared SUSY vacuum must be larger than the Planck scale $M_{\text{Pl}} = 1$, where the supergravity calculation might not be valid. It would be possible that the potential is lifted for $|X| > 1$ by the effect of quantum gravity, the above SUSY vacuum is washed out and the SUSY breaking minimum remains as a global minimum. If the supergravity approximation is valid even for $|X| > 1$ by any reason, we obtain a constraint on the R-breaking parameter ϵ as

$$\epsilon < 2(\sqrt{3} - 1)|f/m|,$$

from the above condition.

We also find a SUSY minimum outside the hyperplane $\phi_i = X_{a \neq 1} = 0$, which is a generalization of eq. (5.11), given by

$$\begin{aligned} |\Phi_B|^2 &= -\frac{2}{\mu_B} \left\{ \mu_B + c + \frac{f^2}{\mu_B - \epsilon m} \left(1 + \frac{\epsilon m}{2(\mu_B - \epsilon m)} \right) \right\} \geq 0, \\ \Phi_{A \neq B} &= K_{A \neq B} = 0, \\ X &= \frac{f}{\mu_B - \epsilon m}, \end{aligned} \tag{6.5}$$

where we assumed the minimal Kähler potential (3.11), and the absence of the higher order terms of X in the superpotential for concreteness. In the limit $\epsilon \rightarrow 0$, this solution is

reduced to (5.11). In contrast to (5.11), the above solution (6.5) does not disappear in all of the parameter region, even after the vacuum energy at the SUSY breaking minimum is set to zero as in eq. (6.2). Such parameter region of μ_B and ϵ allowing the SUSY solution is shown in figure 1. In the shaded region, the SUSY solution (6.5) is not allowed and the SUSY breaking solution does not become metastable due to the R-breaking effect represented by ϵ . Note that we find no allowed region along the $\epsilon = 0$ axis, which corresponds to the case of the solution (5.11).

6.2 Generic case: $g_1(\phi_i) \neq 0$

Finally we introduce nonvanishing $g_1(\phi)$. As in section 5.2, we first consider the case with $m_1^i = 0$ in eq. (5.2). In this case we can still integrate ϕ_i and $X_{a \neq 1}$ by use of $D_{\phi_i} W = D_{X_{a \neq 1}} W = 0$ resulting in $\phi_i = X_{a \neq 1} = 0$.

The remnant of these heavy fields would be the one-loop mass m_X for $X_1 = X$ in eq. (5.13). The effective scalar potential is in the same form as eq. (5.13) but the effective superpotential W in $G = K + \ln |W|^2$ is now replaced by eq. (6.1). For $\epsilon \ll c \sim f \sim m_X \ll 1$ in the unit with $M_{Pl} = 1$, we can obtain a SUSY breaking Minkowski minimum

$$X_{SB} = \frac{2fc}{m_X^2} (1 + \mathcal{O}(\epsilon^2)), \tag{6.6}$$

where the R-breaking constant

$$c = f/\sqrt{3} + \mathcal{O}(f^3/m_X^2; \epsilon^2),$$

is determined by the vanishing vacuum energy condition.

The SUSY ground state in the hyperplane $\phi_i = X_{a \neq 1} = 0$ which originates from the R-breaking parameter ϵ is the same as eq. (6.4), and the above breaking minimum becomes metastable. Unlike (6.2), the SUSY breaking minimum (6.6) is not affected by the R-breaking term at $\mathcal{O}(\epsilon)$ due to the one-loop mass m_X , that is, the SUSY minimum (6.4) is independent of the SUSY breaking minimum (6.6) at this order. There might exist SUSY points analogous to eq. (6.5) outside the hypersurface $\phi_i = X_{a \neq 1} = 0$ also in this case, but the solution would be more complicated due to the nonvanishing h_1^{ij} in eq. (5.2).

Finally we comment about the case with $m_1^i \neq 0$ in eq. (5.2). In this case, as mentioned in section 5.2, the field X_1 has a SUSY mass with the same magnitude as those of ϕ_i 's as shown in eq. (5.12). Then the field X_1 in the field basis (2.12) is no longer special. In this generalized OR model with most general R-breaking terms, the total superpotential would be written as

$$\begin{aligned} W &= fX_1 + \sum_{a=1} g_a(\phi_i)X_a + c(\phi_i) + \frac{1}{2} \sum_{a,b=1} m\epsilon_{ab}(\phi_i)X_aX_b + \dots \\ &= c + fX_1 + \frac{1}{2}\mu_{IJ}\Phi_I\Phi_J + \dots, \end{aligned}$$

where $\Phi_I = (X_a, \phi_i)$, $I = (a, i)$ and the ellipses denote the higher order terms in Φ_I . The SUSY mass matrix μ_{IJ} is given by the R-breaking components, $\mu_{ab} = m\epsilon_{ab}(0)$, $\mu_{ij} = \partial_{\phi_i}\partial_{\phi_j}c(0)$ and the R-symmetric components, $\mu_{ai} = 2\partial_{\phi_i}g_a(0)$. Note that $\mu_{1i} = 2\partial_{\phi_i}g_1(0) =$

$2m_1^i$. After the unitary rotation which makes μ_{IJ} diagonal, the above superpotential takes the form of

$$W = c + fU_{1I}\Phi_I + \frac{1}{2}\mu_I\Phi_I^2 + \dots,$$

where U_{IJ} is the rotation matrix and μ_I represents the eigenvalues of μ_{IJ} . The F-flat conditions, $D_I W = W_I + K_I W = 0$, allow a solution in general and SUSY would not be broken for $m_1^i \sim f$.

7. Conclusion

We considered $N = 1$ global and local supersymmetric models with a continuous global $U(1)_R$ symmetry, and studied the effect of explicit R-symmetry breaking terms in detail.

In global supersymmetric models, based on the argument by ISS, we have shown that some sort of explicit R-symmetry breaking terms can restore SUSY, and the original SUSY breaking vacuum can become metastable when a certain (but not generic) class of explicit R-symmetry breaking terms are added and/or loop effects stabilize the original SUSY breaking minimum.

We have executed similar analyses in R-symmetric supergravity models. First we examined the general argument by NS in supergravity and found that it also holds with local SUSY except for the nontrivial case where the Kähler potential allows solution for the second condition in eq. (3.2). We presented concrete examples of this exception. These models lead to AdS SUSY stationary solutions and associated SUSY breaking vacua with lower vacuum energy. We found the general argument that this class of SUSY solutions corresponds to at best a saddle point, referring to appendix A.

Then, we studied the generalized OR model in supergravity with explicit R-symmetry breaking terms. We analyzed the structure of newly appeared SUSY stationary points as a consequence of the R-breaking effect and classified them. We have shown that these SUSY solutions disappear for type-A breaking terms (4.6), when we tune the R-breaking constant term in the superpotential such that the original SUSY breaking minimum has a vanishing vacuum energy. In this sense, the introduction of explicit R-breaking terms does not always lead to a metastability of the SUSY breaking vacuum. On the other hand, the introduction of type-B breaking terms (4.7) could cause a metastability of SUSY Minkowski minimum. We examined a parameter region which yields metastable vacuum in some concrete examples.

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A. Supersymmetric masses involving R-axion

In this appendix, we show some general results for the SUSY masses for the scalar component of an R-axion multiplet. For this analysis, it is convenient to redefine the R-charged superfield Y by

$$R = \frac{2}{q_Y} \ln Y,$$

where R can be interpreted as the R-axion superfield. (Note that $R = -aT$ in eq. (3.7).) In this basis, the Kähler potential and the superpotential (2.3) is written as

$$\begin{aligned} K &= K(R + \bar{R}, \chi_i, \bar{\chi}_i), \\ W &= e^R \zeta(\chi_i). \end{aligned} \tag{A.1}$$

From eq. (A.1), we find $W^{-1} \partial_Y^m W = 1$ where $m = 1, 2, \dots$, and obtain

$$\begin{aligned} G_{RR} &= K_{RR} + W^{-1} W_{RR} - (W^{-1} W_R)^2 = K_{RR} = K_{R\bar{R}}, \\ G_{\chi_i R} &= K_{\chi_i R} + W^{-1} W_{\chi_i R} - (W^{-1} W_{\chi_i})(W^{-1} W_R) = K_{\chi_i R} = K_{\chi_i \bar{R}}. \end{aligned}$$

Substituting these into the general formulae for the second derivatives at the SUSY point,

$$\begin{aligned} V_{I\bar{J}} \Big|_{D_K W=0} &= e^G (G^{M\bar{N}} G_{MI} G_{\bar{N}\bar{J}} - 2G_{I\bar{J}}), \\ V_{IJ} \Big|_{D_K W=0} &= -e^G G_{IJ}, \end{aligned}$$

we find

$$V_{R\bar{R}} \Big|_{D_K W=0} = V_{RR} \Big|_{D_K W=0} = -K_{R\bar{R}} m_{3/2}^2, \tag{A.2}$$

$$V_{\chi_i \bar{R}} \Big|_{D_K W=0} = V_{\chi_i R} \Big|_{D_K W=0} = -K_{\chi_i \bar{R}} m_{3/2}^2, \tag{A.3}$$

where $m_{3/2}^2 = e^G$ is the gravitino mass square.

From eq. (A.2), the mass squared eigenvalues of $(\text{Re } R, \text{Im } R)$ can be calculated as 0 and $-2m_{3/2}^2$ with the canonical kinetic terms normalized by $K_{R\bar{R}} > 0$. The first massless eigenmode corresponds to the R-axion scalar associated to the spontaneously broken global $U(1)_R$ symmetry. The second negative-definite eigenvalue indicates that the special SUSY solution (3.2) is at best a saddle point solution. Note that the gravitino mass $m_{3/2}$ is nonvanishing at this point and the vacuum energy is negative. We also find from eq. (A.3) that the mixing-mass between R and χ_i is vanishing if the Kähler (kinetic) mixing is vanishing, $K_{\chi_i \bar{R}} = 0$. In this case, the R-axion direction is completely separated from the other fields χ_i , that is, the above mass eigenvalues of R-axion multiplet become exact in this case.

Finally we comment that the second derivatives (A.2) and (A.3) are all vanishing at the SUSY point (3.1) where $m_{3/2} = 0$. In this case, both the real and the imaginary scalar component of R-axion multiplet remain massless. Note that eq. (3.2) may also allow a solution even in this case if ζ is *not* a generic function.

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